THE DIFFERENTIAL CROSS SECTION FOR COMPTON SCATTERING

INTRODUCTION:

Melissinos and Napolitano provide an adequate introduction to this concept in section 9.2 of their text.

The theoretical scattering cross-section for the Compton Interaction can be written (see Refs. 2 and 3):

$$\frac{d\sigma}{d\Omega} = \frac{r_0^2}{2}\left(\frac{1+cos^2(\theta)}{1+\gamma(1-cos\theta)^2}\right) \times \left(1+\frac{\gamma^2(1-cos\theta)^2}{(1+cos^2\theta)[1+\gamma(1-cos\theta)]}\right) \quad (1)$$

where,

$$\gamma = \frac{E_\gamma}{m_0c^2}$$

For the E_γ of 0.662 from ^{137}Cs we arrive at a value of 1.29 for γ. Also,

$$r_0 = 2.82 \times 10^{-13} cm \ \ (classical\ e^-\ radius)$$

Figure 1 shows $E\gamma'$ at θ = 60°. The values of (∑-β) have to be corrected for the intrinsic peak efficiency of the detector. The corrected sum is given by:

$$\left(\frac{\Sigma-\beta}{t}\right)_{corrected} = \frac{1}{\epsilon_p}\left(\frac{\Sigma-\beta}{t}\right)$$

where ε_p is the intrinsic peak efficiency for $E\gamma'$.

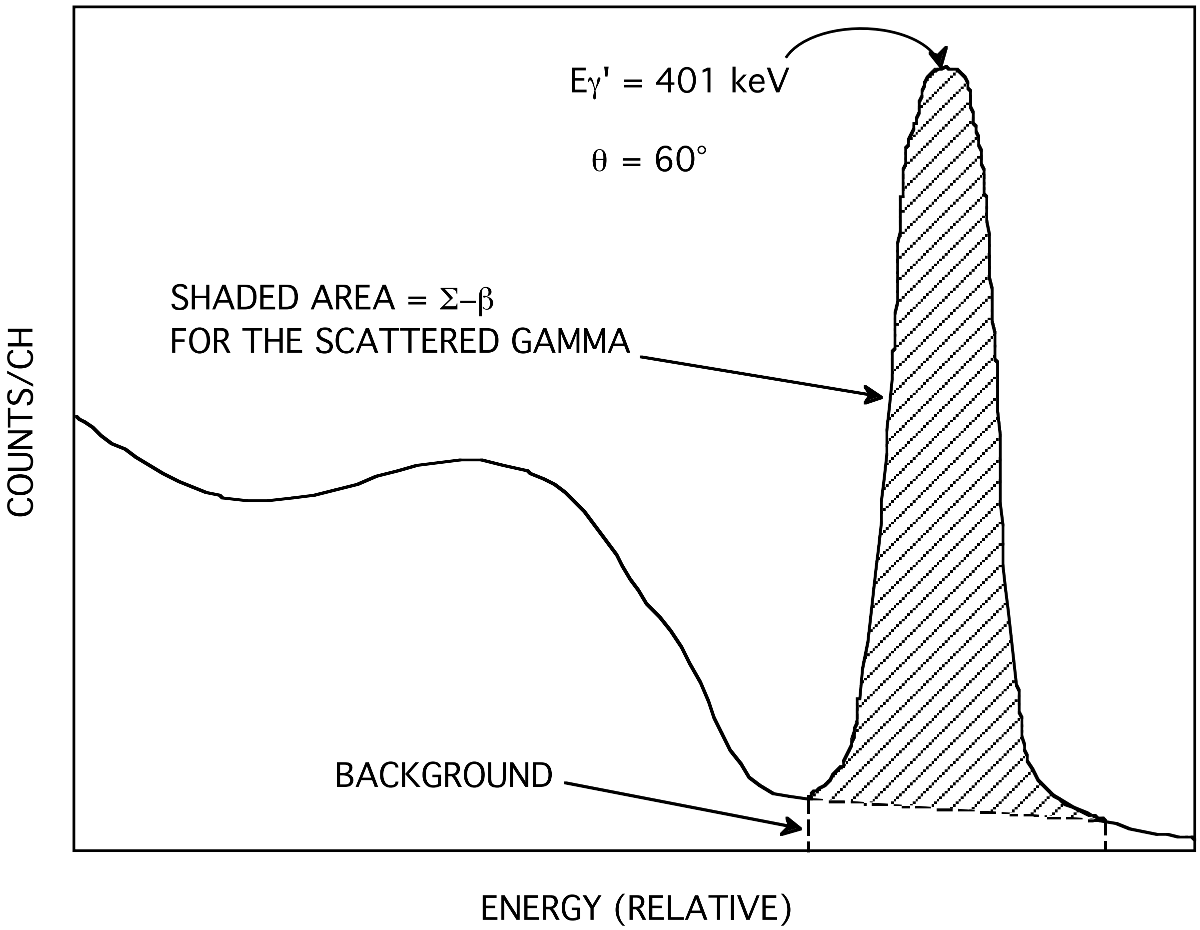

Figure 1. NaI(Tℓ) pulse height spectrum of Compton scattered gammas at q = 60 degrees from ^{137}Cs. (Note: At q = 0 degrees, the scattered peak would have the full energy of the ^{137}Cs source 662 keV.)

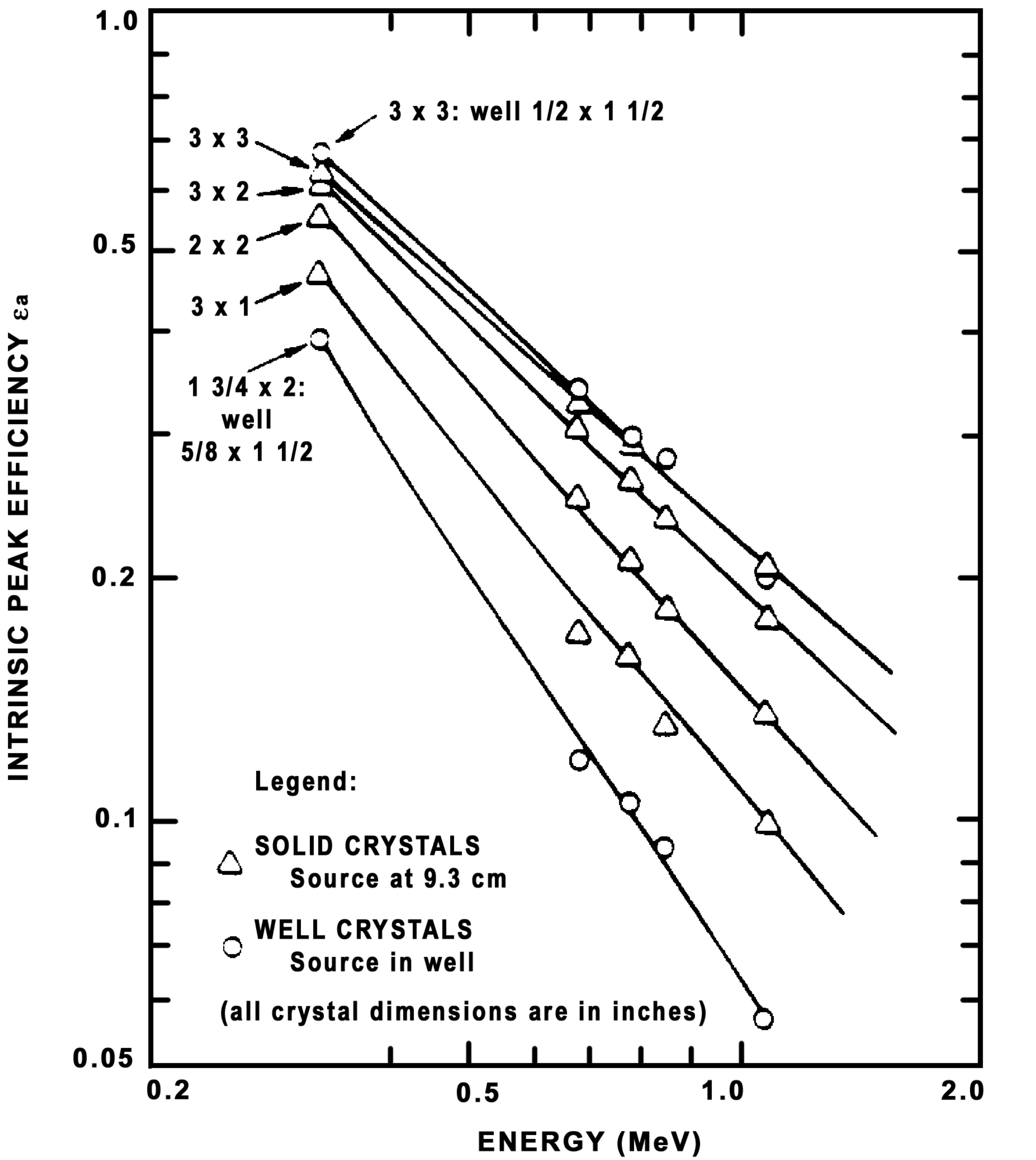

Figure 2. Intrinsic peak efficiency (e_a) for a wide variety of NaI(Tℓ) crystals. The source to detector distance is 9.3 cm (Courtesy of Idaho Operations Office DOE).

For example, the (∑-β) shown in Figure 1 would be divided by the appropriate ε_p for an energy of 401 keV. The measured Compton cross-section is then given by:

$$\frac{d\sigma}{d\Omega}_m = \frac{\left(\frac{\Sigma-\beta}{t}\right)_{corrected}}{n\Delta\Omega\Phi_\gamma} \tag{14}$$

where

n = the number of electrons in the scattering volume .

$$= \frac{\text{(volume) (density of Al) (Avogadros No)}}{\text{(Atomic Weight)}} \tag{15}$$

$\Delta\Omega$ = Solid angle of the NaI (Tℓ) in steradians

$$= \frac{\text{Area of the detector } (\text{cm}^2)}{R_2^2\,(\text{cm}^2)} \tag{16}$$

I_o = The number of incident γ's on the Alminum scatterer per cm ^{2} per S.

$$= \text{The number of } \gamma\text{'s from the source divided by} \tag{17}$$

$$\frac{1}{4\pi R_1^2} \text{ (see Figure 9.4)}$$

NOTE: in Eq. (15), the volume of the aluminum scatterer is given by:

$$V = \pi R_0^2 h \tag{18}$$

where

R_o = .635 cm

h = R_1 sin f_1 (see Figure 9.4)

f_1 = 3.58 degrees

From your experimental data (S-b/t) in Table 9.1 calculate $(ds/dW)_{measure}$ Eq. (14). Enter these experimental points on the theoretical curve 9.6 as shown in the figure. The results should show that the Klein-Nishina theory [Eq. (12)] does a good job of predicting the scattered Compton differential cross section.

References

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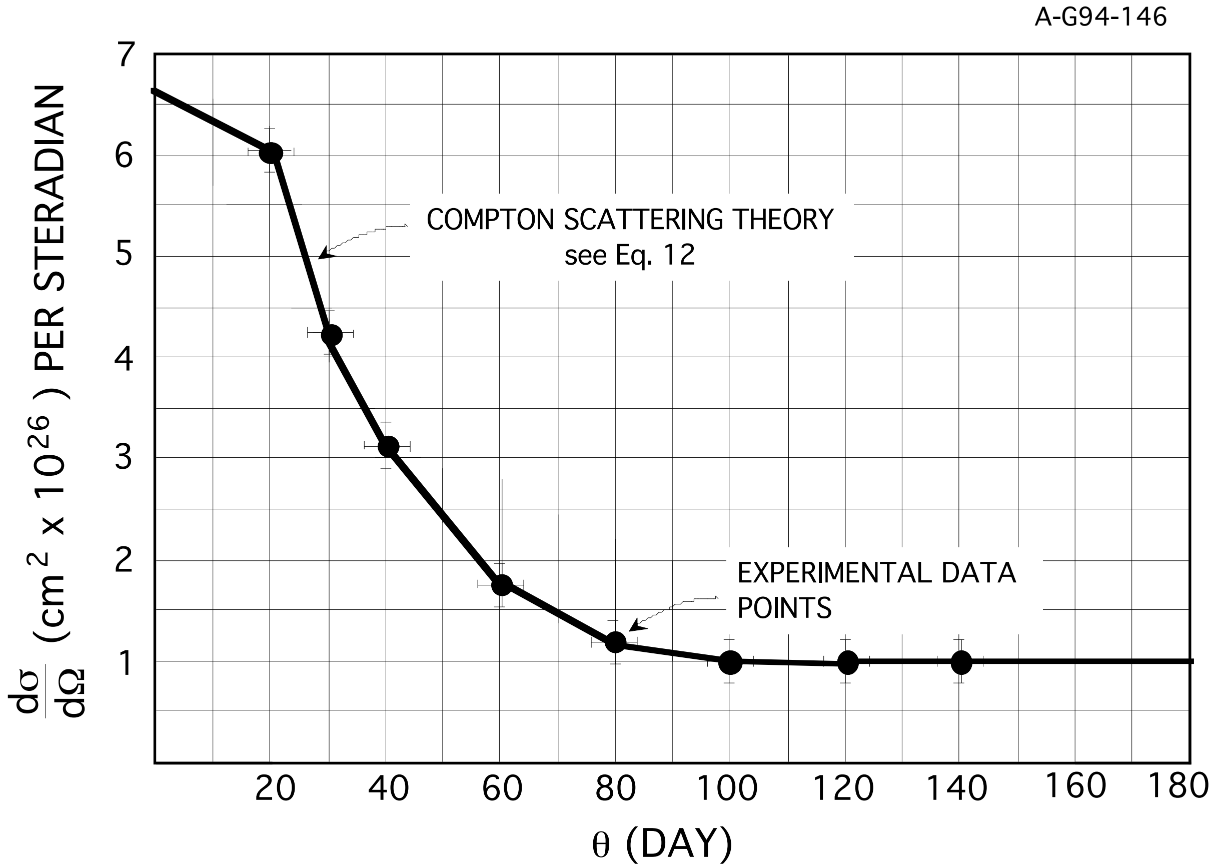

Figure 9.6. Theoretical Compton scattering cross section vs. angle.